Non-adiabatic effect of neutrino oscillation

→ Non adiabatic effect induces transition between $\tilde{\nu}_1$ and $\tilde{\nu}_2$

→ This effect is important only around resonance region.

→ For other regions, the adiabatic approximation can be used

X = Transition probability from $\tilde{\nu}_1$ to $\tilde{\nu}_2$

I) ν_e produced in the solar core goes to survive as ν_e in the non-adiabatic condition, provided no jump occurs, a phenomenon with the probability $(1-X)$

II) ν_e produced in the solar core would have ended as ν_μ, but in the non-adiabatic case might be ended as ν_e if jump occurs with probability X.

$$\Rightarrow \quad P_{\nu_e\nu_e} = (1-X)\, P^{(ad)}_{\nu_e\nu_e} + X\, P^{(ad)}_{\nu_e\nu_\mu}$$

$$= \frac{1}{2}\left(1+\cos 2\tilde{\theta}_0 \cos 2\theta\right)(1-X) + \frac{1}{2}\left(1-\cos 2\tilde{\theta}_0 \cos 2\theta\right) X$$

$$\Rightarrow \quad P_{\nu_e\nu_e} = \frac{1}{2}\left[1+(1-2X)\cos 2\tilde{\theta}_0 \cos 2\theta\right]$$

where, $X = \exp\left(-\gamma_R F\right)$

F is independent of energy E and depends on how n_e varies as x

If variation of n_e with x is linear

F is constant

Evaluation of jump probability X

$$\tilde{E}_\alpha = E - \frac{1}{\sqrt{2}} G_F n_n + \frac{m_\alpha^2}{2E}$$

$$\tilde{m}_{1,2} = \frac{1}{2}\left[\left(m_1^2 + m_2^2 + A\right) \mp \sqrt{\left(\Delta\cos 2\theta - A\right)^2 + \Delta^2 \sin^2 2\theta}\right]$$

In the Landau's method of complex trajectory

$$\ln X = -2\,\mathrm{Im}\left[S_1(t_1, t_*) + S(t_*, t_2)\right]$$

$S_1(t_1, t^*) \longrightarrow$ action for neutrino beam $\tilde{\nu}_1$ from t_1 to transition time t_*

$S_2(t^*, t_2) \longrightarrow$ action for neutrino beam $\tilde{\nu}_2$ from t_* to final time t_2 (when neutrino beam goes out of the non-adiabatic region)

Imaginary parts of the action remains unaffected if $t_1 = t_2 = t_R$

$t_R \rightarrow$ time at which neutrino beam crosses the resonance point

$$\Rightarrow \ln X = -2\,\mathrm{Im}\int_{t_R}^{t_*} dt\,(\tilde{E}_2 - \tilde{E}_1)$$

$$\Rightarrow \ln X = -\frac{1}{E}\,\mathrm{Im}\int_{A_R}^{A^*} \frac{dA}{(dA/dx)} \sqrt{\left(\Delta\cos 2\theta - A\right)^2 + \Delta^2 \sin^2 2\theta}$$

$A_R = \Delta\cos 2\theta$

$A_* \Longrightarrow A$ at transition point $(\tilde{E}_2 = \tilde{E}_1)$

Such transition does not happen for any real A

If $\tilde{E}_2 = \tilde{E}_1$

$$\Rightarrow (\Delta \cos 2\theta - A_*)^2 + \Delta^2 \sin^2 2\theta = 0$$

$$\Rightarrow (\Delta \cos 2\theta - A_* + i\Delta \sin 2\theta)(\Delta \cos 2\theta - A_* - i\Delta \sin 2\theta) = 0$$

$\Rightarrow$ Either $A_* = \Delta \cos 2\theta + i\Delta \sin 2\theta = \Delta e^{2i\theta}$

or $A_* = \Delta \cos 2\theta - i\Delta \sin 2\theta = \Delta e^{-2i\theta}$

$$\Rightarrow A_* = \Delta e^{\pm 2i\theta}$$

If the variation of A as a function of x is linear $\Rightarrow \frac{dA}{dx}$ is constant

$\Rightarrow \frac{dA}{dx}$ can be taken out of integration sign

If $\frac{dA}{dx} > 0$ choose $A_* = \Delta e^{2i\theta}$

< 0 choose $A_* = \Delta e^{-2i\theta}$

Let $a = \dfrac{A - \Delta \cos 2\theta}{\Delta \sin 2\theta}$ ~~$\Rightarrow A = a \sin 2$~~

$\Rightarrow A = a\Delta \sin 2\theta + \Delta \cos 2\theta$ and $dA = \Delta \sin 2\theta \, da$

$A_* = \Delta e^{\pm 2i\theta} \Rightarrow a_* = \pm i$

$\left|\frac{dA}{dx}\right|$ at resonance $\Rightarrow a_*$ is taken at i

Now $\sqrt{(\Delta \cos 2\theta - A)^2 + \Delta^2 \sin^2 2\theta} = \sqrt{a^2+1}\, \Delta \sin 2\theta$

$$\Rightarrow \sqrt{(\Delta \cos 2\theta - A)^2 + \Delta^2 \sin^2 2\theta}\, dA = \sqrt{a^2+1}\, \Delta \sin^2 2\theta \, da$$

$$\ln X = -\frac{\Delta^2 \sin^2 2\theta}{E} \frac{1}{|dA/dx|_R} \, \mathrm{Im} \int_0^i \sqrt{1+a^2}\, da$$

$$\Rightarrow \ln X = -\frac{\Delta^2}{E} \sin^2 2\theta \frac{1}{|dA/dx|_R} \frac{\pi}{4}$$

$$\left|\frac{d\ln A}{dx}\right| = \frac{1}{|A|} \left|\frac{dA}{dx}\right|$$

$$\Rightarrow \frac{1}{|dA/dx|_R} = \frac{1}{A_R \left|\frac{d}{dx}(\ln A)\right|}$$

$$A_R = \Delta \cos 2\theta$$

$$\Rightarrow \ln X = -\frac{\Delta}{E} \frac{\sin^2 2\theta}{\cos 2\theta} \frac{1}{|dA/dx|_R} \frac{\pi}{4}$$

$$\Rightarrow \ln X = -\gamma_R \frac{\pi}{4}$$

$$\Rightarrow X = \exp\left(-\gamma_R \frac{\pi}{4}\right)$$

For exponential fall off near solar density

$$X = \exp\left[-\frac{\pi}{4}\left(1-\tan^2\theta\right)\gamma_R\right]$$

i.e, $F = -\frac{\pi}{4}\left(1-\tan^2\theta\right)$