

Linear and logarithmic scales.

Defining Scale

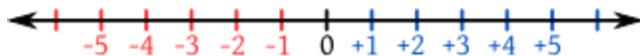
You may have thought of a scale as something to weigh yourself with or the outer layer on the bodies of fish and reptiles. For this lesson, we're using a different definition of a scale. A **scale**, in this sense, is a leveled range of values/numbers from lowest to highest that measures something at regular intervals. A great example is the classic number line that has numbers lined up at consistent intervals along a line.

What Is a Linear Scale?

A **linear scale** is much like the number line described above. The key to this type of scale is that the value between two consecutive points on the line does not change no matter how high or low you are on it.

For instance, on the number line, the distance between the numbers 0 and 1 is 1 unit. The same distance of one unit is between the numbers 100 and 101, or -100 and -101. However you look at it, the distance between the points is constant (unchanging) regardless of the location on the line.

A great way to visualize this is by looking at one of those old school intro to Geometry or mid-level Algebra examples of how to graph a line. One of the properties of a line is that it is the shortest distance between two points. Another is that it has a constant slope. Having a constant slope means that the change in x and y from one point to another point on the line doesn't change.



A standard number line

What Is a Logarithmic Scale?

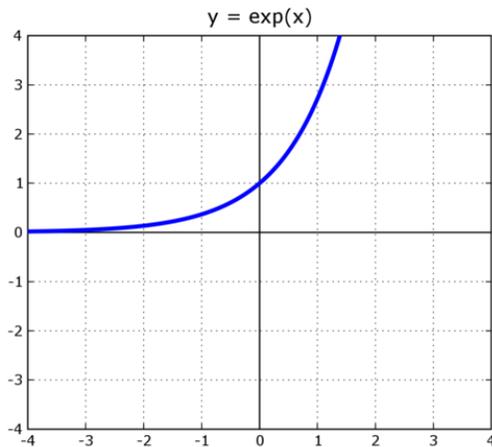
A **logarithmic scale** is much different. On this scale the value between two consecutive points not only changes, but also has a distinct pattern.

- **Logarithms** or 'logs' are based on exponents.
- **Exponents** are the 'little numbers' that are written as superscripts next to a base variable or number. For example, in the expressions $2^3 = 4$, the number 3 is the exponent. These numbers multiply the base by itself a designated amount of times. In 2^3 , the exponent tells us that 2 should be multiplied by itself 3 times: $2^3 = 2 \times 2 \times 2 = 8$.

Imagine that we need to measure a really large quantity of something. Maybe minerals in soil, molecules in air, or the intensity of sound waves, for example. Sometimes we need to create a simplified scale where each step represents a large number of units and also increases/decreases by a certain factor.

If a scientist needs to measure billions or even trillions of molecules, they might just make a logarithmic scale with each number (i.e. from 0 to 1) increase representing an increase by a factor of 10. That would mean that going from 0 to 1 means increasing 10 units, and going from 0 to 2 means increasing 100 units, because $10^2 = 100$. Numbers on a logarithmic scale are representative of a factor increase in real units.

A great way to visualize this is by looking at the graph of an exponential function. One of the properties shown in the example below is that, as x increases, y increases 'exponentially' or by a greater quantity for every additional unit of x .



This exponential line increases by greater and greater quantities for every unit increase in x

Application and Use

Linear Scales

Linear scales are very good for measurements in the real world. Your standard school ruler is a perfect example. Your 10 centimeters are the same 10 centimeters anywhere in the world. It's a simple exercise in basic counting and each unit has an equal, in this case, length.

Logarithmic scales

There are two main reasons to use logarithmic scales in charts and graphs. The first is to respond to skewness towards large values; i.e., cases in which one or a few points are much larger than the bulk of the data. The second is to show percent change or multiplicative factors. First I will review what we mean by logarithms. Then I will provide more detail about each of these reasons and give examples.

To refresh your memory of school math, logs are just another way of writing exponential equations, one that allows you to separate the exponent on one side of the equation. The

equation $2^4 = 16$ can be rewritten as $\log_2 16 = 4$ and pronounced "log to the base 2 of 16 is 4." It is helpful to remember that the log is the exponent, in this case, "4". The equation $y = \log_b(x)$ means that y is the power or exponent that b is raised to in order to get x . The common base for logarithmic scales is the base 10. However, other bases are also useful. While a base of ten is useful when the data range over several orders of magnitude, a base of two is useful when the data have a smaller range.